# DESCRIPTION OF THE ANOMALOUS RAYLEIGH-TAYLOR INSTABILITY ON THE BASIS OF THE MODEL OF DYNAMICS OF A THREE-VELOCITY THREE-TEMPERATURE MIXTURE 

G. A. Ruev, ${ }^{1}$ A. V. Fedorov, ${ }^{2}$ and V. M. Fomin ${ }^{2}$

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#### Abstract

A problem of evolution of the Rayleigh-Taylor instability on a sinusoidally perturbed interface in the field of artificial gravity is studied on the basis of the model of a three-velocity three-temperature mixture of gases. If the upper part of the mixture consists of the light and heavy gases (the heavy gas is heavier than the pure gas at the bottom, but the mixture as a whole is lighter than the pure gas), it is demonstrated that the growth of perturbations with formation of mixture jets penetrating into the pure gas is observed in a certain range of concentrations of the heavy gas.


Key words: Rayleigh-Taylor instability, gas-dynamics of a three-velocity three-temperature mixture.

Introduction. A mathematical model of a three-velocity three-temperature mixture of gases is proposed to study the evolution of the Rayleigh-Taylor instability at the interface of a mixture of two gases on one side and the third (pure) gas on the other side. The accelerated motion of the mixing layer separating two gases with different densities is accompanied by the development of instability, which arises if the presence of the light medium leads to acceleration of motion of the heavy medium or if the heavy medium is located above the light medium in the gravity field. This type of instability is called the Rayleigh-Taylor instability [1]. Traditionally, the mixing layer is considered as a surface of density discontinuity, i.e., as a contact discontinuity. The influence of the processes of mutual penetration of the gases was ignored in numerical simulations of the evolution of the Rayleigh-Taylor instability based on the Euler equations. Therefore, it seems of interest to study this problem on the basis of equations for a multi-velocity multi-temperature mixture of gases, where each species has its own velocity and temperature. Such an approach allows us to describe both the processes of mutual penetration of the gases and the evolution of the mixing layer arising on the interface. The results of research of the instability evolution on the interface in binary mixtures were described in detail in $[2-4]$. In the present paper, we describe the results of investigations of the development of the Rayleigh-Taylor instability arising on the interface of a three-species mixture of gases, where the mixture of the light and heavy gases is located above the less heavy gas in the field of artificial gravity. The density of the mixture located above the interface is smaller than the density of the gas below the interface. From the viewpoint of the classical gas dynamics, such a mixture is stable to comparatively small perturbations [1]. The emergence of convection in the gravity field was observed in such a mixture in experiments [5-7]. Obviously, this phenomenon cannot be adequately described within the framework of the one-velocity and even two-velocity approximation. It is necessary to use the equations of gas dynamics of a three-velocity threetemperature mixture.

Formulation of the Problem. Let us consider the evolution of perturbations on the interface between a mixture of two gases and a pure gas. Initially, this system is in equilibrium under the action of a constant

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Fig. 1. Formulation of the problem of the evolution of perturbations on the interface between two media.
acceleration. The upper part of the domain contains the mixture of the light gas No. 1 and the heavy gas No. 2; the lower part contains the gas No. 3, which is less heavy than the gas No. 2 (Fig. 1). The interface is perturbed by sinusoidal perturbations whose shape is described by the equation $x_{0}(y)=a_{0}(1-\cos (\theta y))$, where $a_{0}$ is the perturbation amplitude, $\theta=2 \pi / \lambda$ is the wavenumber, and $\lambda$ is the length of the perturbation wave.

The motion in the region containing all three species is described by the equations of gas dynamics of a three-velocity three-temperature mixture [8]

$$
\begin{align*}
& \frac{\partial \boldsymbol{U}_{i}}{\partial t}+\frac{\partial \boldsymbol{F}_{i}^{(1)}}{\partial x}+\frac{\partial \boldsymbol{F}_{i}^{(2)}}{\partial y}=\boldsymbol{W}_{i} ;  \tag{1}\\
& \boldsymbol{U}_{i}=\left[\begin{array}{c}
\rho_{i} \\
\rho_{i} u_{i} \\
\rho_{i} v_{i} \\
E_{i}
\end{array}\right], \quad \boldsymbol{F}_{i}^{(1)}=\left[\begin{array}{c}
\rho_{i} u_{i} \\
\rho_{i} u_{i}^{2}+p_{i} \\
\rho_{i} u_{i} v_{i} \\
u_{i} E_{i}+p_{i} u_{i}
\end{array}\right], \quad \boldsymbol{F}_{i}^{(2)}=\left[\begin{array}{c}
\rho_{i} v_{i} \\
\rho_{i} v_{i} u_{i} \\
\rho_{i} v_{i}^{2}+p_{i} \\
v_{i} E_{i}+p_{i} v_{i}
\end{array}\right], \\
& \boldsymbol{W}_{i}=\left[\begin{array}{c}
0 \\
\sum_{j=1}^{3} K_{i j \neq i)}\left(u_{j}-u_{i}\right)-g \rho_{i} \\
\sum_{j=1(j \neq i)}^{3} K_{i j}\left(v_{j}-v_{i}\right) \\
\sum_{j=1}^{3} K_{i j} u_{i}\left(u_{j}-u_{i}\right)+\sum_{j=1}^{3} K_{(j \neq i)}^{3} K_{i j} v_{i}\left(v_{j}-v_{i}\right)+ \\
+\sum_{j=1}^{3} \beta_{i j} K_{i j}\left(\left(u_{j}-u_{i}\right)^{2}+\left(v_{j}-v_{i}\right)^{2}\right)+\sum_{j=1}^{3} q_{(j \neq i)} K_{i j} K_{i j}\left(T_{j}-T_{i}\right)
\end{array}\right], \\
& p_{i}=k n_{i} T_{i}, \quad E_{i}=\rho_{i}\left(e_{i}+\frac{u_{i}^{2}+v_{i}^{2}}{2}\right), \quad e_{i}=\frac{k T_{i}}{\gamma_{i}-1}, \quad \rho_{i}=m_{i} n_{i}, \quad i, j=1,2,3(i \neq j) .
\end{align*}
$$

Here $\rho_{i}$ is the density, $u_{i}$ and $v_{i}$ are the velocity components, $e_{i}$ is the internal energy, $p_{i}$ is the pressure, $T_{i}$ is the temperature, $m_{i}$ is the mass of the molecule, $n_{i}$ is the number density of molecules of the $i$ th species, $x$ and $y$ are the Cartesian coordinates, $t$ is the time, $k$ is the Boltzmann constant, $K_{i j}=16 \rho_{i} \rho_{j} \Omega_{i j}^{(1,1)} /\left(3\left(m_{i}+m_{j}\right)\right)$, $\Omega_{i j}^{(1,1)}$ is the collision integral, $\beta_{i j}=m_{i} T_{i} /\left(m_{i} T_{i}+m_{j} T_{j}\right), q_{i j}=3 m_{i} K /\left(m_{i}+m_{j}\right)$, and $\gamma_{i}$ is the ratio of specific heats.

The interaction potential for solid spheres is described by the relation

$$
K_{i j}=\frac{16}{3} \frac{\rho_{i} \rho_{j}}{m_{i} m_{j}} \sqrt{\frac{k \pi}{2}} \sqrt{\frac{T_{i}}{m_{i}}+\frac{T_{j}}{m_{j}}} \sigma_{i j}, \quad \sigma_{i j}=\frac{\sigma_{i}+\sigma_{j}}{2}
$$

( $\sigma_{i}$ is the diameter of the molecule of the $i$ th gas).
For a one-temperature mixture, we have $K_{i j}=x_{i} x_{j} p / D_{i j}$, where $D_{i j}$ is the diffusion coefficient of the $i$ th and $j$ th gases.

For small (or zero) concentrations of one of the three gases (e.g., gas No. 1), the equations of gas dynamics of a two-velocity two-temperature mixture of the gas Nos. 2 and 3 are used, and the parameters of the gas No. 1 are determined from the relations

$$
\begin{gathered}
\frac{\partial n_{1}}{\partial t}+\frac{\partial n_{1} u_{1}}{\partial x}+\frac{\partial n_{1} v_{1}}{\partial y}=0 \\
u_{1}=\alpha_{2} u_{2}+\alpha_{3} u_{3}, \quad v_{1}=\alpha_{2} v_{2}+\alpha_{3} v_{3}, \quad T_{1}=\alpha_{2} T_{2}+\alpha_{3} T_{3}
\end{gathered}
$$

For small (or zero) concentrations of two of the three gases (e.g., gas Nos. 1 and 2), the Euler equations are used for the pure gas No. 3, and the parameters of the gas Nos. 1 and 2 are determined from the relations

$$
\begin{gathered}
\frac{\partial n_{j}}{\partial t}+\frac{\partial n_{j} u_{j}}{\partial x}+\frac{\partial n_{j} v_{j}}{\partial y}=0 \\
u_{j}=u_{i}, \quad v_{j}=v_{i}, \quad T_{j}=T_{i}, \quad j=1,2, \quad i=3
\end{gathered}
$$

The transition from one system of equations to another is performed under the condition that the greater of the values of the molar and mass fractions of the $i$ th gas is smaller than $0.1 \%$ :

$$
\max \left(x_{i}, \alpha_{i}\right)<0.1 \%, \quad x_{i}=\frac{n_{i}}{n_{1}+n_{2}+n_{3}}, \quad \alpha_{i}=\frac{\rho_{i}}{\rho_{1}+\rho_{2}+\rho_{3}}
$$

System (1) is solved by the method of flux vector splitting. For the solution to be monotonic in regions with high gradients, the order of approximation was reduced by a minmod limiter used for constructing total variation diminishing (TVD) schemes. Implicit approximation of the right sides of the equations of system (1) was used, which allowed us to avoid enhancing the restriction on the time step imposed by the Courant condition. Details of the method can be found in $[2-4]$.

Formation of the Diffusion Mixing Region. Before passing to the analysis of computation results, we consider the problem of diffusion mixing of a three-species mixture, because the processes that occur during the formation of the diffusion layer can generate conditions for instability development. It was noted [5-7] that the reason for the emergence of convective instability is a nonmonotonic profile of density of the mixture in the diffusion region. Let there be a channel with a partition. The volume on the right of the partition is filled by a mixture of the gas No. 1 with the gas No. 2 uniformly distributed in the gas No. 1 ; the volume on the left of the partition is filled by the pure gas No. 3. At the initial time, the partition is removed, and diffusion mixing begins.

Let us consider a one-dimensional approximation and assume that $K_{i j} \gg 1$, i.e., the times of velocity and temperature relaxation are small. We introduce the mean molar velocities and temperatures: $u=x_{1} u_{1}+x_{2} u_{2}$ and $T=x_{1} T_{1}+x_{2} T_{2}$. Then, in the zero approximation, we obtain

$$
\begin{aligned}
& \frac{\partial n}{\partial t}+\frac{\partial n u}{\partial x}=0, \quad \rho \frac{\partial u}{\partial t}+\rho u \frac{\partial u}{\partial x}+\frac{\partial p}{\partial x}=0, \quad \rho c_{v} \frac{\partial T}{\partial t}+\rho c_{v} u \frac{\partial T}{\partial x}+p \frac{\partial p}{\partial x}=0, \\
& n=n_{1}+n_{2}+n_{3}, \quad p=p_{1}+p_{2}+p_{3} .
\end{aligned}
$$

The solution of this system has the form $u=0, p=$ const, $T=$ const, and $n=$ const.
In the first approximation in terms of $1 / K_{i j}$, we obtain the Stefan-Maxwell relations from the equations of conservation of momentum for the first and third species of the mixture:

$$
\begin{equation*}
\frac{\partial x_{1}}{\partial x}=\frac{x_{1} x_{2}}{D_{12}}\left(u_{2}-u_{1}\right)+\frac{x_{1} x_{3}}{D_{13}}\left(u_{3}-u_{1}\right), \quad \frac{\partial x_{3}}{\partial x}=\frac{x_{1} x_{3}}{D_{13}}\left(u_{1}-u_{3}\right)+\frac{x_{2} x_{3}}{D_{23}}\left(u_{2}-u_{3}\right) . \tag{2}
\end{equation*}
$$

As $n=$ const, the equations of continuity yield

$$
\begin{equation*}
\frac{\partial x_{1}}{\partial t}=-\frac{\partial x_{1} u_{1}}{\partial x}, \quad \frac{\partial x_{3}}{\partial t}=-\frac{\partial x_{3} u_{3}}{\partial x} \tag{3}
\end{equation*}
$$

Expressing $u_{i}$ from Eq. (2) with allowance for the fact that $u=x_{1} u_{1}+x_{2} u_{2}+x_{3} u_{3}=0$, we obtain the equations that describe the diffusion processes from Eq. (3):

$$
\begin{array}{rlrl}
\frac{\partial x_{1}}{\partial t} & =D_{13} \frac{\partial}{\partial x}\left(f_{1} \frac{\partial x_{1}}{\partial x}+f_{2} \frac{\partial x_{3}}{\partial x}\right), & \frac{\partial x_{3}}{\partial t} & =D_{13} \frac{\partial}{\partial x}\left(f_{3} \frac{\partial x_{1}}{\partial x}+f_{4} \frac{\partial x_{3}}{\partial x}\right) \\
f_{1} & =\frac{\tau_{1}\left(1+\left(\tau_{2}-1\right) x_{1}\right)}{1+\left(\tau_{2}-1\right) x_{1}+\left(\tau_{1}-1\right) x_{3}}, & f_{2} & =\frac{\tau_{2}\left(\tau_{1}-1\right) x_{1}}{1+\left(\tau_{2}-1\right) x_{1}+\left(\tau_{1}-1\right) x_{3}} \\
f_{3} & =\frac{\tau_{1}\left(\tau_{2}-1\right) x_{3}}{1+\left(\tau_{2}-1\right) x_{1}+\left(\tau_{1}-1\right) x_{3}}, & f_{4} & =\frac{\tau_{2}\left(1+\left(\tau_{1}-1\right) x_{3}\right)}{1+\left(\tau_{2}-1\right) x_{1}+\left(\tau_{1}-1\right) x_{3}}  \tag{4}\\
\tau_{1}=D_{12} / D_{13}, & \tau_{2} & =D_{23} / D_{13}
\end{array}
$$

We have to find the solution of system (4) satisfying the initial conditions

$$
\begin{equation*}
x_{1}=0, \quad x_{3}=1 \quad \text { at } \quad x<0, \quad x_{1}=1-x_{2}^{0}, \quad x_{3}=0 \quad \text { at } \quad x>0 \tag{5}
\end{equation*}
$$

Here $x_{2}^{0}$ is the molar fraction of the gas No. 2 in the mixture of the gas Nos. 1 and 2 at the initial time.
Let us consider some asymptotic approximations for problem (4), (5).
Let $\tau_{1}=\tau_{2}=1$. Then, the solution of system (4), (5) takes the form

$$
x_{1}=\frac{1-x_{2}^{0}}{2}(1+\operatorname{erf}(\xi)), \quad x_{3}=\frac{1}{2}(1-\operatorname{erf}(\xi)), \quad \xi=\frac{x}{2 \sqrt{D_{13} t}}
$$

The expression for density of the mixture is written as

$$
\rho=m_{1} x_{1}+m_{2}\left(1-x_{1}-x_{3}\right)+m_{3} x_{3}, \quad \frac{\partial \rho}{\partial x}=-\frac{1}{2} \frac{\mathrm{e}^{-\xi^{2}}}{\sqrt{\pi t D_{13}}}\left(m_{3}-m_{1}-x_{2}^{0}\left(m_{2}-m_{1}\right)\right)
$$

Obviously, the profile of density of the mixture has a monotonic character.
Let $\tau_{1} \ll 1$ and $\tau_{2}=1$. In this case, the diffusion coefficient of the gas Nos. 1 and 2 is small. Then, in the zero approximation, Eq. (4) yields

$$
\begin{equation*}
\frac{\partial x_{3}}{\partial t}=D_{13} \frac{\partial^{2} x_{3}}{\partial x^{2}}, \quad \frac{\partial x_{1}}{\partial t}+D_{13} \frac{\partial}{\partial x}\left(\frac{x_{1}}{1-x_{3}} \frac{\partial x_{3}}{\partial x}\right)=0 \tag{6}
\end{equation*}
$$

The solution of system (6) satisfying the initial conditions (5) has the form

$$
x_{3}=(1-\operatorname{erf}(\xi)) / 2, \quad x_{1}=\left(1-x_{2}^{0}\right)\left(1-x_{3}\right), \quad x_{2}=1-x_{1}-x_{3} .
$$

In this case, the profile of density of the mixture is also monotonic.
Let $\tau_{1}=1$ and $\tau_{2} \ll 1$. Then, the diffusion coefficient of the gas Nos. 2 and 3 is small. In this case, the mixture consists of the light gas No. 1 and the heavy gas No. 2; the gas No. 3 is also heavy. In the zero approximation, Eq. (4) yields

$$
\begin{equation*}
\frac{\partial x_{1}}{\partial t}=D_{13} \frac{\partial^{2} x_{1}}{\partial x^{2}}, \quad \frac{\partial x_{3}}{\partial t}+D_{13} \frac{\partial}{\partial x}\left(\frac{x_{3}}{1-x_{1}} \frac{\partial x_{1}}{\partial x}\right)=0 \tag{7}
\end{equation*}
$$

The solution of system (7) satisfying the initial conditions (5) has the form

$$
x_{1}=\frac{1-x_{2}^{0}}{2}(1+\operatorname{erf}(\xi)), \quad x_{3}=\left\{\begin{array}{cc}
1-x_{1}, & \xi<\xi_{0},  \tag{8}\\
0, & \xi>\xi_{0},
\end{array} \quad x_{2}=1-x_{1}-x_{3}\right.
$$

The value of $\xi_{0}$ is found by solving the nonlinear equation

$$
\begin{equation*}
2 \xi_{0}=\left.\left(\frac{1}{1-x_{1}} \frac{d x_{1}}{d \xi}\right)\right|_{\xi=\xi_{0}} \tag{9}
\end{equation*}
$$

Figure 2 shows the density of the mixture as a function of the parameter $\xi-\xi_{0}$ for different values of the initial concentration of the gas No. 2 in the mixture $x_{2}^{0}$, which was calculated by Eqs. (8) and (9). The gas Nos. 1,2 , and 3 were chosen to be helium, xenon, and argon, respectively. It is seen that the density of the mixture decreases in the beginning. Then, in a narrow zone $\Delta \xi$ of the order $\tau_{2}$, the density increases (in the approximation used, in a jumplike manner) to a certain value higher than the density of the mixture on the right of the interface, and then the density decreases and tends to the latter value. It follows from this approximation that the profile of


Fig. 2. Density of a three-species mixture (helium-xenon-argon) versus the parameter $\xi-\xi_{0}$, calculated by Eqs. (8) and (9): $x_{2}^{0}=0.01$ (1), 0.25 (2), and 0.5 (3).
density of the mixture can be nonmonotonic. In turn, the existence of a segment with increasing density in the case with a lighter medium on the right of the interface can be the reason for instability development.

The full equations (2) were calculated for the helium-xenon mixture initially located on the right of the interface and argon located on the left of the interface. The model of solid spheres was used to calculate the diffusion coefficients. The calculation parameters had the following values: $\sigma_{2} / \sigma_{1}=4.94 / 2.19, \sigma_{3} / \sigma_{1}=3.66 / 2.19$, $m_{2} / m_{1}=32.8$, and $m_{3} / m_{1}=10$.

Figure 3 shows the density of the mixture as a function of the coordinate $x$ at different times after the beginning of the diffusion process for three initial concentrations of xenon: $x_{2}^{0}=10,35$, and $25 \%$. It is seen that the profile is nonmonotonic for $x_{2}^{0} \geqslant 10 \%$. The case with $x_{2}^{0}=25 \%$ is of the greatest interest. At the initial time, the density of the mixture on the right of the interface is smaller than the density of the gas (argon) on the left of the interface. Owing to diffusion, a segment with increasing density appears on the density profile. This nonmonotonicity arises as a consequence of the so-called diffusion "gate" of the heavy species (peak in the xenon concentration profile in Fig. 4) [5-7].

Rayleigh-Taylor Instability. Let us consider the evolution of perturbations on the interface in the problem formulation illustrated in Fig. 1. At the initial time, the upper part is filled by a mixture of helium and xenon, and the lower part is filled by argon. The entire system is thermodynamically equilibrium. Under the assumption that all gases are at rest and have an identical temperature, system (1) yields the conditions of static equilibrium for such a mixture in the field of artificial gravity with acceleration $g$ :

$$
\begin{align*}
& T_{1}=T_{2}=T_{3}=T=\mathrm{const}, \quad u_{i}=v_{i}=0, \\
& x>0: \quad \frac{d n_{i}}{d x}=-\frac{g m_{i}}{k T} n_{i}, \quad i=1,2, \quad n_{3}=0,  \tag{10}\\
& x<0: \quad \frac{d n_{3}}{d x}=-\frac{g m_{3}}{k T} n_{3}, \quad n_{i}=0, \quad i=1,2, \\
& x=0: \quad n_{1}+n_{2}=n_{3}, \quad n_{2} /\left(n_{1}+n_{2}\right)=x_{2}^{0} .
\end{align*}
$$

The pressure and temperature on the upper boundary $x_{k}$ were $p=p_{0}=0.5 \mathrm{~atm}$ and $T=300 \mathrm{~K}$. Small sinusoidal perturbations were introduced on the interface at the initial time.

The calculations were performed in a rectangular domain $\left[x_{n}, x_{k} ; 0, \lambda / 2\right]$. Conditions (10) were satisfied on the lower boundary $x_{n}$ and on the upper boundary $x_{k}$. Symmetry conditions were imposed on the side boundaries.

Figure 5 shows the isolines of the molar fractions of xenon and helium at different times (the initial concentration of xenon was $x_{2}^{0}=25 \%$, the acceleration corresponding to the artificial gravity force was $g=10^{6} \mathrm{~m} / \mathrm{sec}^{2}$, the perturbation amplitude was $a_{0}=5 \mathrm{~mm}$, and the perturbation wavelength was $\lambda=36 \mathrm{~mm}$ ). Typical indications of the development of the Taylor instability are observed, in particular, the formation of a jet penetrating from the


Fig. 3. Density of a three-species mixture (helium-xenon-argon) versus the coordinate $x$, calculated by Eqs. (2) at different times and for different initial concentrations of xenon: $x_{2}^{0}=10$ (a), 35 (b), $25 \%$ (c); $t=0$ (1), 0.1 (2), and $1 \sec$ (3).


Fig. 4. Concentrations of the species in the helium-xenon-argon mixture versus the coordinate $x$ for $x_{2}^{0}=25 \%$ at different times: $t=0$ (1), 0.1 (2), and $1 \mathrm{sec}(3)$; the solid, dashed, and dotted curves refer to xenon, argon, and helium, respectively;


Fig. 5. Isolines of the molar fractions of xenon (a, c, e, and g) and helium (b, d, f, and h) at different times: $t=0$ ( a and b), 500 (c and d), 1000 (e and f), and $2000 \mu \mathrm{sec}$ ( g and h).

$a$



Fig. 6. Isolines of the concentration of the binary mixture (helium-xenon) in a three-species mixture (helium-xenonargon) calculated on the basis of the two-velocity model for $t=500$ (a) and $2000 \mu$ sec (b).
initially lighter medium into the heavier medium. The mixture becomes enriched by the heavy gas in the valleys, which is responsible for the emergence of instability.

Calculations were also performed for the case where the mixture in the upper part is a single gas with an effective mass of molecules $m_{\text {mix }}$, mean ratio of specific heats $\gamma_{\text {mix }}$, and mean diameter of molecules $\sigma_{\text {mix }}$ :

$$
\begin{gathered}
m_{\operatorname{mix}}=x_{1} m_{1}+x_{2} m_{2}, \quad \gamma_{\operatorname{mix}}=\frac{\alpha_{1} c_{1 v} \gamma_{1}+\alpha_{2} c_{2 v} \gamma_{2}}{\alpha_{1} c_{1 v}+\alpha_{2} c_{2 v}} \\
\sigma_{\operatorname{mix}}=\frac{x_{1} \sqrt{1 / m_{1}+1 / m_{2}}\left(\sigma_{1}+\sigma_{3}\right) / 2+x_{2} \sqrt{1 / m_{2}+1 / m_{3}}\left(\sigma_{2}+\sigma_{3}\right) / 2}{\sqrt{1 /\left(x_{1} m_{1}+x_{2} m_{2}\right)+1 / m_{3}}}
\end{gathered}
$$

The formula for $\sigma_{\text {mix }}$ follows from system (1) via a limiting transition to the two-velocity model, where the mixture of helium and xenon is considered as a one-velocity mixture.

Figure 6 shows the isolines of the helium and xenon concentrations in the helium-xenon-argon mixture. It is seen that the initial perturbations do not develop further. A similar result is obtained in calculations based on the equations of gas dynamics for a one-species gas.

Figure 7 shows the total width of the mixing region of the gases as a function of time for different values of the parameters. The total width of the mixing region was determined as the distance between the point with the maximum coordinate $x$ where the argon concentration was smaller than $5 \%$ and the point with the minimum coordinate $x$ where the argon concentration was higher than $95 \%$. Curves 5 and 7 refer to the case where the mixture in the upper part is a single gas; the remaining curves were calculated for a three-species mixture of the gases. It is seen that reduction of the initial concentration of xenon $x_{2}^{0}$ ensures a lower growth rate of perturbations; the growth of perturbation ceases altogether at low values of the concentration (curve 6). At $x_{2}^{0}=35$ and $50 \%$ (curves 3 and 4 , respectively), the initial density of the mixture is higher than the initial density of argon, i.e., the classical development of the Rayleigh-Taylor instability is observed. Reduction of the perturbation amplitude leads to lower growth rates of perturbations (curves 1 and 2). At high concentrations of xenon, the evolution of perturbations is similar to that observed in a two-species mixture (curves 4 and 5). Curves 1 and 7 refer to the case where the initial density of the mixture in the upper part is lower than the initial density of the gas in the lower part. The three-species model (curve 1) predicts faster growth of the mixing region than the two-species model (curve 7).

Conclusions. In this work, the mathematical model of a multi-velocity multi-temperature mixture of gases [8] is used to study the Rayleigh-Taylor instability in a mixture of three gases.

The formation of a diffusion mixing layer is described within the framework of a simplified mathematical model. It is demonstrated that the profile of density of the mixture in the diffusion region can be nonmonotonic, which may be the reason for instability development in the case with the lighter medium in the upper part of the domain in the gravity field.
$L, \mathrm{~mm}$


Fig. 7. Total width of the mixing region of the gases versus time: curves 1-4 and 6 were calculated by the three-species model with $x_{2}^{0}=25 \%$ and $a_{0}=5 \mathrm{~mm}(1), x_{2}^{0}=25 \%$ and $a_{0}=3 \mathrm{~mm}(2), x_{2}^{0}=35 \%$ and $a_{0}=5 \mathrm{~mm}(3), x_{2}^{0}=50 \%$ and $a_{0}=5 \mathrm{~mm}(4)$, and $x_{2}^{0}=10 \%$ and $a_{0}=5 \mathrm{~mm}(6)$; curves 5 and 7 were calculated by the two-species model with $x_{2}^{0}=50 \%$ and $a_{0}=5 \mathrm{~mm}$ (5) and $x_{2}^{0}=25 \%$ and $a_{0}=5 \mathrm{~mm}$ (7).

The problem of evolution of the Rayleigh-Taylor instability on a sinusoidally perturbed interface between the mixture and the pure gas in the field of artificial gravity is studied. It is shown that instability evolution is possible if the density of the mixture is smaller than the density of the pure gas located in the lower part of the region.

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[^0]:    ${ }^{1}$ Novosibirsk State University of Architecture and Civil Engineering, Novosibirsk 630008. ${ }^{2}$ Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; ruev@sibstrin.ru; fedorov@itam.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 50, No. 1, pp. 58-67, January-February, 2009. Original article submitted November 28, 2007; revision submitted January 16, 2008.

